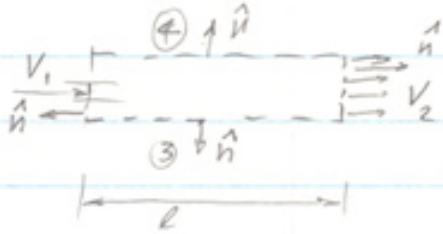
UE Fluids F7 solution Fall 07



a) $\oint \rho \vec{V} \cdot \hat{n}\, dA = \int_{①} + \int_{②} + \int_{③} + \int_{④}$

$\int_{①} \rho \vec{V} \cdot \hat{n}\, dA = -\rho V_1 h/8$

$\int_{②} \rho \vec{V} \cdot \hat{n}\, dA = \rho V_2 h$

$\int_{③} \rho \vec{V} \cdot \hat{n}\, dA = 0$

$+ \int_{④} \rho \vec{V} \cdot \hat{n}\, dA = 0$

$\oint \rho \vec{V} \cdot \hat{n}\, dA = -\rho V_1 h/8 + \rho V_2 h = 0 \Rightarrow \boxed{V_2 = \frac{1}{8} V_1}$

b)

$\int_{①} [\rho(\vec{V}\cdot\hat{n})\vec{V} + p\hat{n}]\, dA = -\rho V_1 \frac{h}{8} \cdot V_1 \hat{i} - p_1 h \hat{i}$

$\int_{②} [\rho(\vec{V}\cdot\hat{n})\vec{V} + p\hat{n}]\, dA = \rho V_2 h\, V_2 \hat{i} + p_2 h \hat{i}$

$\int_{③} [\rho(\vec{V}\cdot\hat{n})\vec{V} + p\hat{n}]\, dA = 0 - p_3 \ell \hat{j}$

$+ \int_{④} [\rho(\vec{V}\cdot\hat{n})\vec{V} + p\hat{n}]\, dA = 0 + p_4 \ell \hat{j}$

$\oint [\rho(\vec{V}\cdot\hat{n})\vec{V} + p\hat{n}]\, dA = \left[-\rho V_1^2 h/8 + \rho V_2^2 h + (p_2 - p_1)h\right]\hat{i} + \left[(p_4 - p_3)\ell\right]\hat{j} = \vec{0}$

Taking only x component, and using $V_2 = \frac{1}{8} V_1$:

$$-\rho V_1^2 \frac{h}{8} + \rho V_1^2 \frac{h}{64} + (p_2 - p_1)h = 0$$

$$\boxed{p_2 - p_1 = \rho V_1^2 \cdot \frac{7}{64} > 0}$$

Since $p_2 - p_1 > 0$, then $p_2 > p_1$

Pressure increases downstream.